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Trapped ions and the shielding of dust particles in low-density non-equilibrium plasma of glow discharge

Gennady Sukhinin[1,2,4], **Alexander Fedoseev**[1], **Sergei Antipov**[3], **Oleg Petrov**[3] **and Vladimir Fortov**[3]

[1] Institute of Thermophysics SB RAS, Ave. Lavrentyev, 1, Novosibirsk 630090, Russia
[2] Novosibirsk State University, Pirogova Str., 2, Novosibirsk, 630090, Russia
[3] Joint Institute for High Temperatures RAS, Izhorskaya 13/19, Moscow 127412, Russia

E-mail: sukhinin@itp.nsc.ru

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Abstract
A new model for the formation of trapped ions around a negatively charged dust particle immersed in low-density non-equilibrium plasma of gas discharge is presented. It is shown that the ionic coat leads to a shielding of the proper charge of the dust particle. In experiments it is only possible to detect the effective charge of a dust particle that is equal to the difference between the proper charge of the particle and the charge of trapped ions.

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(Some figures in this article are in colour only in the electronic version)

A dust particle charge plays a great role in complex plasma, and it is important to know that for understanding different physical processes in dusty plasma [1]. It determines the interaction of the particle with other dust grains, ions and electrons in the surrounding plasma, and with external electric fields. In their turn, the plasma parameters determine the dust particle charge. Even a single dust particle immersed in low-temperature plasma presents an example of a strongly coupled Coulomb system. The orbital motion limited (OML) theory for spherical grains in low-density plasma is often applied to obtain the charge of dust particles [2]. This approach deals with collisionless electron and ion trajectories in the vicinity of a small probe or dust particle and only the laws of energy and angular momentum conservation are used to calculate electron and ion fluxes to the surface of grain. However, Goree [3] in 1992 showed that ions can lose energy in rare collisions with atoms and become trapped in finite orbits by the electric field of a charged particle. The problem of trapped ions was studied by Lampe in

[4] Author to whom correspondence should be addressed.

papers [4, 5] with the help of the analytical method and in a number of other papers [6–9] with the help of different numerical methods. In all these papers, the limitations of OML theory were considered and the influence of collisions was investigated.

In this paper, we present a new alternative model (equation (7)) for collisionless conditions for the formation of trapped or bound ions around the dust particle in low-pressure plasma when the newly formed trapped ions have the time to make several rotations around the particle before the next collision. Only in such rare collisional regimes it is possible to separate free and trapped ions, and trapped ions screen the proper charge of a dust particle in the same way as orbital electrons screen the nucleus in an atom [5]. This statement is investigated through comparison with experimental data on the charge of a dust particle [10, 11] measured partly in rare collisional plasma. The obtained results are extrapolated to the collisional region of the experimental data where they should be corrected.

Below we consider the model of the formation of an ion coat around a dust particle with radius r_0 that consists of trapped ions captured on finite orbits as a result of rare charge exchange collisions of neutral atoms with free ions falling to the particle from ambient plasma. When far from the particle, ions have an isotropic Maxwellian distribution function. On approaching the charged particle, ions accelerate in a self-consistent electric potential $U(r)/e$ of a particle immersed in plasma and they undergo rare elastic collisions with neutral atoms of a parent gas. The consideration of elastic collisions is a very difficult problem, which requires the solution of the Boltzmann equation or should be solved with the help of Monte Carlo methods (if differential cross sections of ion-neutral atom are known). However, elastic collisions of ions in the parent noble gas have a sharp peak of backscattering when the colliding ion and atom exchange their velocities. In this paper, for mathematical simplification we consider only such resonant charge exchange ion-atom collisions. Moreover, we assume that the collision frequency, $\nu = N_g\sigma_{res}\mathrm{v}_{\text{i-a}}$, is independent of relative ion-atom velocity $\mathrm{v}_{\text{i-a}}$. Such assumptions were also made in [5].

Radial distribution of free ions, $N_{if}(r)$, has the form [5–7]

$$N_{if}(r) = \frac{N_0}{2}\left\{\int_0^\infty \mathrm{d}\varepsilon\, f_M(\varepsilon)\left[\sqrt{1-U(r)/\varepsilon} + \sqrt{1-\frac{r_0^2}{r^2}}\sqrt{1-E_0(r)/\varepsilon}\right]\right\}, \tag{1}$$

where N_0 is the density of the ambient plasma, ε is the ion kinetic energy, $f_M(\varepsilon) = (2/\sqrt{\pi}T^{3/2})\exp(-\varepsilon/T)\varepsilon^{1/2}$ is the Maxwellian energy distribution function of both neutral atoms and free ions in the ambient plasma (far from the charged dust particle), $T_i = T$ is the temperature of atoms and ions, $U(r)/e$ is a self-consistent electric potential of a particle immersed into plasma. In this paper, we use definitions for radial-dependent functions

$$E_0(r) = \left[r^2U(r) - r_0^2U(r_0)\right]/\left(r^2 - r_0^2\right), \qquad E_m(r) = r_0^2(U(r) - U(r_0))/\left(r^2 - r_0^2\right). \tag{2}$$

After the resonant charge exchange collision of free ion with a neutral thermal atom at distance R from the charged particle, the atom is transformed into an ion with velocity $\vec{\mathrm{v}}$ characterized by angle θ with respect to the direction of the radius vector $\vec{R}$. The newly formed ion can be trapped by a dust particle with probability $P_{tr}(r)$, fall to the particle surface with probability $P_{fall}(r)$, or become free with probability $P_{free}(r)$ depending on the values of kinetic energy ε and orbital momentum $J = \mathrm{M}vR\sin\theta$. To be trapped, the newly formed ions should have negative energy $E_t(R) = \varepsilon + U(R) < 0$ (we used the assumption that effective potential $U_{eff}(r) = U(r) + \varepsilon R^2\sin^2\theta/r^2$ has no maxima, see [5]) and cannot reach the particle surface, i.e. the minimal distance from the trapped ion to the particle center, r_{min}, satisfies the relation:

$r_{\min} = r_0$. This condition gives the range of angles $\theta_{\min} < \theta < \pi - \theta_{\min}$ of ion velocity after the charge exchange collision with an atom in which the ion becomes trapped:

$$\sin^2\theta_{\min}(R,\varepsilon) = \frac{r_0^2}{R^2}\left(1 + \frac{U(R) - U(r_0)}{\varepsilon}\right). \tag{3}$$

The trapped ion kinetic energy satisfies the condition: $0 < E_m(r) < \varepsilon < -U(r)$. The average probability $P_{\mathrm{tr}}(R)$ is

$$P_{\mathrm{tr}}(r) = \int_{E_m(r)}^{-U(r)} \mathrm{d}\varepsilon\, f_M(\varepsilon) \int_{\theta_{\min}}^{\pi-\theta_{\min}} \frac{\sin\theta}{2}\,\mathrm{d}\theta = \sqrt{1 - \frac{r_0^2}{R^2}}\cdot\frac{2}{\sqrt{\pi}} \int_{E_m(r))/T}^{-U(r)/T} \sqrt{y - \frac{E_m(r)}{T}}\,\mathrm{e}^{-y}\,\mathrm{d}y. \tag{4}$$

From equation (4), it can be concluded that trapped ions can be formed in the region from r_0 to R_0 only, where R_0 is defined from the relation $r_0^2 U(r_0) = R_0^2 U(R_0)$.

In the same way, we introduce probability $P_{\mathrm{fall}}(r)$ that after a charge exchange collision an ion falls on the particle, and probability $P_{\mathrm{free}}(r)$ that the ion acquires positive total energy and therefore runs away to infinity (or for some interval of angles θ, falls on the particle). These probabilities are equal to

$$P_{\mathrm{fall}}(r) = \frac{2}{\sqrt{\pi}} \int_0^{-U(r)/T} \mathrm{d}y\,\mathrm{e}^{-y}\sqrt{y} - \sqrt{1 - \frac{r_0^2}{r^2}}\cdot\frac{2}{\sqrt{\pi}} \int_{E_m(r)/T}^{-U(r)/T} \sqrt{y - \frac{E_m(r)}{T}}\,\mathrm{e}^{-y}\,\mathrm{d}y, \tag{5}$$

where we take into account the fact that ions with small energy ($\varepsilon < E_m(r)$) fall on the particle irrespective of their angular momentum, and

$$P_{\mathrm{free}}(r) = \frac{2}{\sqrt{\pi}} \int_{-U(R)/T}^{\infty} \sqrt{y}\exp(-y)\,\mathrm{d}y. \tag{6}$$

It can be easily verified that $P_{\mathrm{tr}}(r) + P_{\mathrm{free}}(r) + P_{\mathrm{fall}}(r) = 1$.

In unit time as a result of charge exchange collisions of free ions with neutral atoms (with collisional frequency ν), trapped ions are created at point R and move around a charged particle contributing to the density of trapped ions $N_{\mathrm{tr}}(r)$ at different points along their finite trajectories. This contribution in the layer dr is proportional to the ratio of ion residence time in dr, $\mathrm{d}t(R, r, \varepsilon, \theta) = \mathrm{d}r/\mathrm{v_r}(R, r, \varepsilon, \theta)$, ($\mathrm{v_r}(R, r, \varepsilon, \theta)$ is the radial velocity of the trapped ion formed at point R and transferred to point r) to time $T_{\mathrm{tr}}(R, \varepsilon, \theta)$ of an ion moving from $r_{\min}(R, \varepsilon, \theta)$ to $r_{\max}(R, \varepsilon, \theta)$. Thus, we can obtain the balance equation for the creation and loss of trapped ions:

$$\nu N_{\mathrm{tr}}(r)(P_{\mathrm{fall}}(r) + P_{\mathrm{free}}(r)) = \nu \int_{r_0}^{R_0} \mathrm{d}R\,\frac{R^2}{r^2} N_{\mathrm{if}}(R) \int_{E_m(R)}^{-U(R)} \mathrm{d}\varepsilon f_M(\varepsilon) \int_{\pi-\theta_m}^{\theta_m} \mathrm{d}\theta\,\frac{\sin\theta}{2}$$
$$\times \frac{\Theta\left(1 - \frac{R^2}{r^2}\sin^2\theta + \frac{U(R)-U(r)}{\varepsilon}\right)}{\sqrt{1 - \frac{R^2}{r^2}\sin^2\theta + \frac{U(R)-U(r)}{\varepsilon}}} \cdot \frac{1}{\int_{r_{\min}(R,\varepsilon,\theta)}^{r_{\max}(R,\varepsilon,\theta)} \frac{\mathrm{d}r'}{\sqrt{1 - \frac{R^2}{r'^2}\sin^2\theta + \frac{U(R)-U(r')}{\varepsilon}}}}, \tag{7}$$

In this equation, we introduce the Heaviside step function, $\Theta(x) = 1, x > 0$; $\Theta(x) = 0, x < 0$, which ensures the calculation of integrals only in the accessible region of parameters. The left side of this equation takes into account the fact that trapped ions disappear after a subsequent collision with thermal atoms as a result of falling to the surface of the particle or their acquiring a positive total energy in the collision. The balance equation (7) for trapped and free ions are valid only in the first approximation for low-density conditions, $\nu\overline{T_{\mathrm{tr}}(R)} < 1$, when the division of ions into two groups (free and trapped ions) can be justified ($\overline{T_{\mathrm{tr}}(R)}$ is an averaged characteristic time of trapped ion rotation around charged particle). In a collisional case, the

Vlasov–Boltzmann equation or the Bhatnagar–Gross–Krook (BGK) equation for the velocity distribution of ions should be solved (see the recent paper [8]). The collisional case was also studied with the help of particle-in-cell calculations by Hutchinson and Patacchini [9]. It should be stressed that the model presented by equation (7) is consistent with the low-frequency limit of the solution of the BGK equation obtained in [8].

Integrating equation (7) over r from r_0 to R_0, we can receive an obvious total balance of trapped ions creation and loss in the whole region around a charged particle:

$$4\pi\nu\int_{r_0}^{R_0} \mathrm{d}R\, R^2 N_{\mathrm{tr}}(R)(P_{\mathrm{fall}}(R)+P_{\mathrm{free}}(R)) = 4\pi\nu\int_{r_0}^{R_0}\mathrm{d}R\, R^2 N_{\mathrm{if}}(R)P_{\mathrm{tr}}(R). \tag{8}$$

The calculation of equation (7) is rather difficult due to the singularities of the integral kernel in the right side of the balance equation, and due to the necessity of calculating the self-consistent potential $U(r)$ that require an iterative approach. In this paper, we consider a simplified version of equation (7). It is seen that singularities of the kernel in equation (7) are most pronounced at $R = r$ and $\theta = \pi/2$. It means that a trapped ion at its trajectory spends most time near the remote turning point. Thus we can obtain an approximate form of the balance equation

$$N_{\mathrm{if}}(r)P_{\mathrm{tr}}(r) = N_{\mathrm{tr}}(r)(P_{\mathrm{fall}}(r)+P_{\mathrm{free}}(r)), \tag{9}$$

which is consistent with a total balance equation (8). Possibly, this approximation leads to a narrower radial distribution of trapped ions than the exact distribution obtained from equation (7).

However, as is seen in equation (8), it does not change the total number of trapped ions.

All probabilities in equation (9) are functions of electric potential $U(r)$, which must be determined self-consistently from the solution of the Poisson equation. Taking into account the spherical symmetry of volume charge distribution, we can obtain

$$U(r) = -\frac{e^2 Z_0}{r} + \frac{4\pi e^2}{r}\int_{r_0}^{r}\mathrm{d}x\, x^2\Delta N(x) + 4\pi e^2\int_{r}^{\infty}\mathrm{d}x\, x\Delta N(x), \tag{10}$$

where $\Delta N(r) = N_{if}(r) + N_{\mathrm{tr}}(r) - N_e(r)$ is the density of volume charge of ions and electrons at point r. The volume charge and the charge of trapped ions in the space between the particle and the sphere of radius r are equal to

$$Q(r) = 4\pi e\int_{r_0}^{r}\mathrm{d}r\, r^2(N_{\mathrm{if}}(r)+N_{\mathrm{tr}}(r)-N_e(r)), \qquad Q_{\mathrm{tr}}(r) = 4\pi e\int_{r_0}^{r}\mathrm{d}r\, r^2 N_{\mathrm{tr}}(r). \tag{11}$$

To obtain distributions $U(r)$, $N_{\mathrm{tr}}(r)$ and $Q_{\mathrm{tr}}(r)$, the following iterative procedure was used. At the initial step the dust particle charge number Z_0 was chosen for the given discharge parameters (gas and ambient plasma densities, N_g, N_0, electric field, E_z, ion temperature, T_i) according to the OML model. Assuming that the initial electric potential radial distribution is Debye–Hückel, $U^0(r) = -e^2 Z_0\exp(-r/\lambda_i)/r$, where $\lambda_i = (T_i/4\pi e^2 N_0)^{1/2}$, we calculated the radial distributions of all probabilities (4)–(6) and ion number densities $N_{\mathrm{if}}(r)$, $N_{\mathrm{tr}}(r)$. Then a new distribution of a self-consistent electric potential $U(r)$ was found with the help of the Poisson equation. The final electric potential and volume charge distributions and the collisional ion flux to the particle surface, I_{tr}, were found using the iterative method. The collisional ion flux of the trapped ions was calculated with the help of the formula

$$I_{\mathrm{tr}} = 4\pi\nu\int_{r_0}^{R_0} P_{\mathrm{fall}}(r)N_{\mathrm{tr}}(r)r^2\,\mathrm{d}r, \tag{12}$$

which is valid in the first approximation for low-pressure plasma conditions. The total ion flux to the particle is the sum of a free ion flux (OML model) and a collisional ion flux, $I_i = I_{\mathrm{if}} + I_{\mathrm{tr}}$. Equating the total ion flux to the electron flux, I_e, we can find a new dust particle charge. For

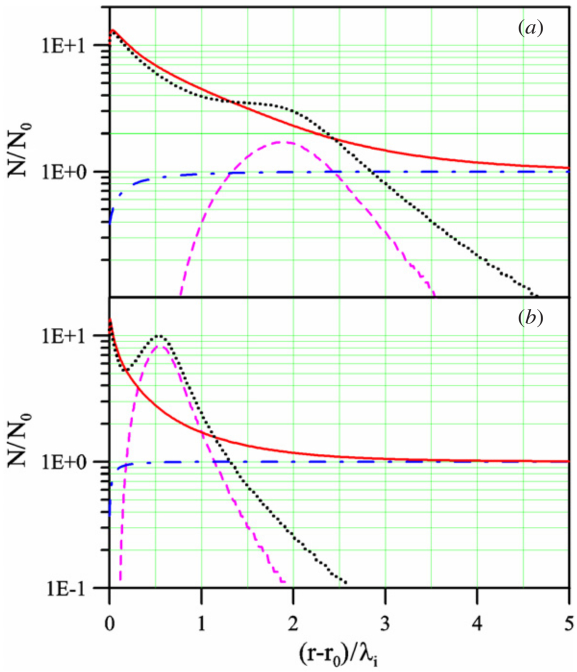

Figure 1. Distributions of free ions (solid lines), $N_{\mathrm{if}}(r)$; trapped ions (dashed lines), $N_{\mathrm{tr}}(r)$; electrons (dash-dotted lines), $N_e(r)$; and total space charge (dotted lines), $\Delta N(r)$. Ion temperature, $T =$ 300 K. (*a*). Debye–Hückel length, $\lambda_i/r_0 = 10$; $\tau = 333$. (*b*) $\lambda_i/r_0 = 66$, $\tau = 333$.

this value of a particle charge, the iterative procedure described above was repeated. The final values of the dust particle charge, the distributions of charged particles and electric potential do not depend neither on the choice of the initial value of Z_0 nor on the initial electric potential distribution $U_0(r)$.

The developed model permits us to calculate $N_{\mathrm{if}}(r)$, $U(r)$ and $Q_{\mathrm{tr}}(r)$ for parameters of low-temperature plasma when the condition $r_0 < \lambda_i < 1/N_g\sigma_{\mathrm{res}}$ is fulfilled. In figure 1, the radial distributions of trapped ions N_{tr}, free ions N_{if} and electrons N_e and the total volume charge ΔN are presented for two values of λ_i for the given $\tau = |Z_0|e^2/r_0T = 333$. It is seen that the trapped ion density $N_{\mathrm{tr}}(r)$ (and the probability $P_{\mathrm{tr}}(r)$) has the maximum value at some distance from the charged particle. For distances beyond this maximum the behavior of $N_{\mathrm{tr}}(r)$ is in full agreement with the results of Lampe [5] and calculations of [8, 9] for $\nu\rightarrow 0$. At small distances, $N_{\mathrm{tr}}(r)$ decreases (possibly due to the approximation made in equation(9)), which contradicts the conclusion made in paper [5]. However, this fact does not influence the value of the total charge of trapped ions, $Q_{\mathrm{tr}}(r\rightarrow\infty)$, which is equal to 40–60% of the proper charge of the dust particle, $|eZ_0|$, depending on λ_i. This result is in agreement with the prediction of Lampe [5]. The calculation of the volume charge $Q(r)$ shows that at infinity the total volume charge is equal to the charge of a particle, eZ_0, i.e. $Q(r\rightarrow\infty) = |eZ_0|$, with great accuracy irrespective of the chosen initial potential $U_0(r)$ and probabilities $P_{\mathrm{tr}}(r)$.

From formula (10), it is seen that radial distribution of electric potential $U(r)$ has a finite jump from value $U_0 = -e^2Z_0/r_0$ to $U(r_0) = U_0 + 4\pi e^2\int_{r_o}^{\infty}\mathrm{d}r\, r\Delta N(r)$. This fact is important for the calculation of self-consistent distributions of electric potential and ion densities. In figure 2, a radial distribution of electric potential, $U(r)$, Coulomb potential of the

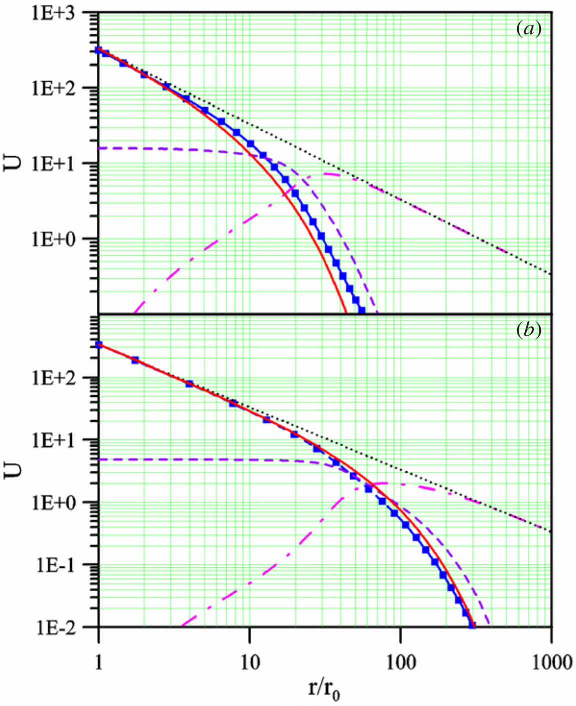

Figure 2. Radial distribution of self-consistent potential $U(r)$ (squares). Coulomb potential of the particle (dotted lines). Initial Debye–Hückel potential curves (solid lines). The radial dependences of the first and second integrals in equation (10) are presented by dashed dot and dashed lines. (*a*) Debye–Hückel length, $\lambda_i/r_0 = 10$; $\tau = 333$; (*b*) $\lambda_i/r_0 = 66$, $\tau = 333$.

particle, initial Debye–Hückel potential curves, the radial dependences of the first and second integrals in equation (10) are presented for the same conditions as in figure 1. It is seen that the second integral in equation (10) is responsible for the shielding of a particle charge in plasma. In particular, trapped ions provide a strong screening of the proper charge of the dust particle, and the resulting potential differs only slightly from the Debye–Hückel potential. It should be mentioned that this result contradicts the radial distribution of the potential obtained in recent papers [8] and [13], where the potential has Coulomb-like asymptotic. Such behavior of the potential at large distances from a charged particle immersed in *plasma* means that there is some sphere in which the total charge of a particle, ions and electrons is not compensated. If this is the case, then plasma outside this sphere will redistribute and screen this residual charge. Under low collision frequencies, in [8, 13] this residual charge is about 1% of the particle charge. However, in [8] the accuracy of numerical calculations was estimated to be 3%. In paper [13] the Coulomb-like asymptotic for a point-like particle was obtained analytically with the help of several approximations. Thus, it is necessary to investigate this problem more carefully due to its importance for understanding the processes occurring in dusty plasma.

In figure 3, the calculated dependence of a particle charge on neon pressure in the range $p = 20$–150 Pa are presented for conditions in a glow discharge with electric field $E = 2.1\text{V cm}^{-1}$ [10, 11]. In these experiments, the condition $\nu\overline{T_{\text{tr}}(R)} < 1$ is fulfilled at least for the low-pressure part of experimental data. The solid line represents the calculations using conventional OML theory with the non-equilibrium electron energy distribution function obtained from the Boltzmann equation [12], $Z_{\text{OML}}(p)$. The dashed line is the calculations of a

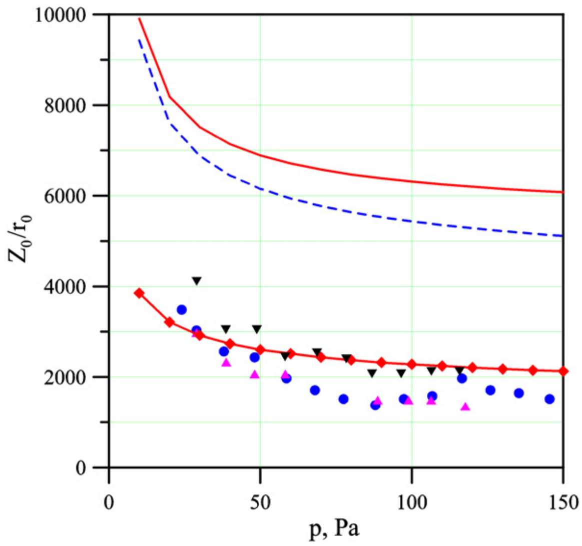

Figure 3. Dependence of particle charge Z_0 normalized to particle radius r_0 on gas pressure: result of the OML model, $Z_{OML}(p)$ (solid line); self-consistent solution for $Z_0(p)$ (dashed line); dust particle effective charge $Z_{eff}(p)$ (–◆–◆–◆–); symbols are experimental data [10, 11] for the particles with different radii ($r_0 = 0.6$, 1.0 and 1.3 μm).

self-consistent particle charge number, $Z_0(p)$, obtained with the help of the presented model. Experimental results [10, 11] are shown by symbols. It is seen that self-consistent particle charge number Z_0 for low-pressure conditions of [10, 11] is only slightly smaller than the charge number calculated with the help of the OML model, Z_{OML}. The difference between experimentally obtained and calculated particle charges for low-pressure conditions cannot be explained in terms of the collisional ion flux only.

In low-density complex plasma, a negatively charged dust particle exists with its ionic coat. The charged particle is bound by an electric field with trapped ions in the same way as the atomic nucleus is bound with orbital electrons [5–7]. The electrostatic force acting on a dust particle with the ionic coat is proportional to electric field strength and the effective charge of the particle, eZ_{eff}. In figure 3, the dependence $eZ_{eff} = eZ_0 - Q_{tr}(r \to \infty)$ on neon pressure was shown by a solid line with squares. The values of an effective particle charge are in fairly good agreement with experimental data [10, 11]. It should be stressed that this curve was obtained without any fitting parameters. The increase of collision frequency in plasma leads to an increase of collision flux of ions to the particle, which reduces the proper particle charge. Simultaneously, the trapped ion coat is gradually destroyed due to collisions of trapped ions along their finite trajectories. These two effects partly compensate each other.

We can conclude that in low-pressure experimental investigations based on the electrostatic interaction of two dust particles or the interaction of a charged dust particle with an external electric field, only an effective charge of a dusty particle rather than a proper charge of a particle can be found, and the screening provided by trapped ions should be taken into account.

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